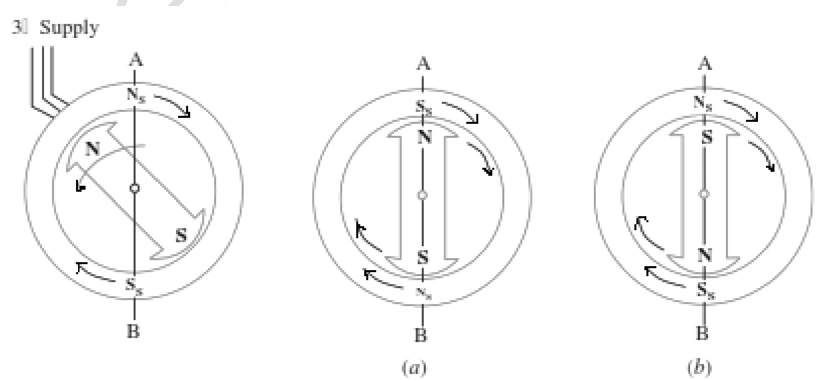
UNIT II SYNCHRONOUS MOTOR

Principle of operation – Torque equation – Operation on infinite bus bars - V and Inverted V curves – Power input and power developed equations – Starting methods – Current loci for constant power input, constant excitation and constant power developed-Hunting – natural frequency of oscillations – damper windings- synchronous condenser.

- If a three phase supply is given to the stator of a three phase alternator, it can work as a motor.
- As it is driven at synchronous speed, it is called synchronous generator.
- So if alternator is run as a motor, it will rotate at a synchronous speed. Such a device which converts an electrical energy into a mechanical energy running at synchronous speed is called synchronous motor.
- Synchronous motor works only at synchronous speed and cannot work at a speed other than the synchronous speed.

PRINCIPLE OF OPERATION (Why Synchronous Motor is not self starting?)

- Synchronous motor works on the principle of the magnetic locking.
- When a 3 phase winding is fed by a 3-phase supply, then a magnetic flux of constant magnitude but ***rotating*** at synchronous speed is produced.
- Consider a two-pole stator of Fig., in which are shown two stator poles (marked N_S and S_S) rotating at synchronous speed, say, in clockwise direction.
- With the rotor position as shown, suppose the stator poles are at that instant situated at points *A* and *B*. The two similar poles, *N* (of rotor) and N_S (of stator) as well as *S* and S_S will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.
- But half a period later, stator poles, having rotated around, interchange their positions *i.e.* N_S is at point *B* and S_S at point *A*. Under these conditions, N_S attracts *S* and S_S attracts *N*. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction).
- Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing *i.e.*, in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction.
- Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that it remains stationary.



- Now, consider the condition shown in Fig. (*a*). The stator and rotor poles are attracting each other.
- Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig. (*b*).
- Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque *i.e*., clockwise torque, as shown in Fig.

Procedure to Start a Synchronous Motor

1. Give a three phase a.c. supply to a three phase winding. This will produce rotating magnetic field rotating at synchronous speed N_s r.p.m.
2. Then drive the rotor by some external means like diesel engine in the direction of rotating magnetic field, at a speed very near or equal to synchronous speed.

3. Switch on the d.c. supply given to the rotor which will produce rotor poles. Now there are two fields one is rotating magnetic field produced by stator while the other is produced by rotor which is physically rotated almost at the same speed as that of rotating magnetic field.

4. At a particular instant, both the fields get magnetically locked. The stator field pulls rotor field into synchronism. Then the external device used to rotate rotor can be removed. But rotor will continue to rotate at the same speed as that of rotating magnetic field i.e. N_s due to magnetic locking.

Methods of Starting Synchronous Motor

The various methods to start the synchronous motor are,

1. Using pony motors 2. Using damper winding. 3. as a slip ring induction motor 4. Using small d.c. machine coupled to it.

1. Using Pony Motors

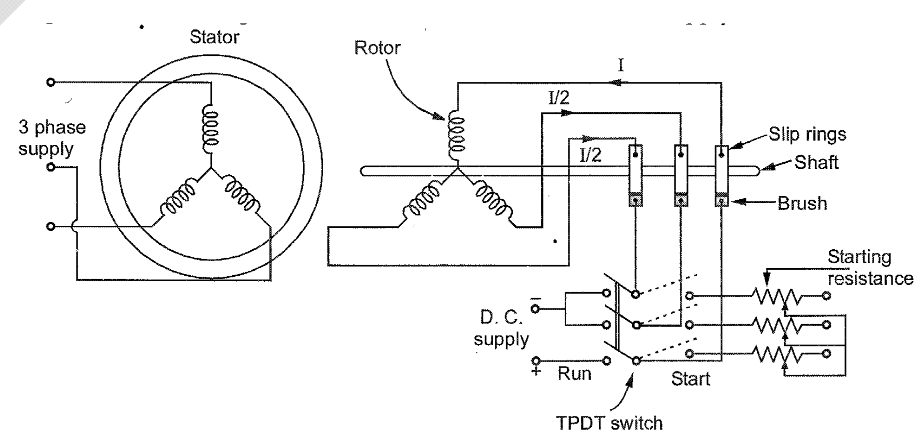
- In this method, the rotor is brought to the synchronous speed with the help of some external device like small induction motor. Such an external device is called Pony Motor'.
- Once the rotor attains the synchronous speed, the d.c. excitation to the rotor is switched on. Once the synchronism is established pony motor is decoupled. The motor then continues to rotate as a synchronous motor.

2. Using Damper Winding

- In a synchronous motor, in addition to the normal field winding, the additional winding consisting of copper bars placed in the slots in the pole faces.
- The bars are short circuited with the help of end rings. Such an additional winding on the rotor is called damper winding.
- This winding as short circuited, acts as a squirrel cage rotor winding of an induction motor.
- Once the stator is excited by a three phase supply, the motor starts rotating as an induction motor at sub synchronous speed. Then d.c. supply is given to the field winding.
- At a particular instant motor gets pulled into synchronism and starts rotating at a synchronous speed.
- As rotor rotates at synchronous speed, the relative motion between damper winding and the rotating magnetic field is zero.
- Hence when motor is running as synchronous motor, there cannot be any induced e.m.f. in the damper winding. So damper winding is active only at start, to run the motor as an induction motor at start. Afterwards it is out of the circuit.
- As damper winding is short circuited and motor gets started as induction motor, it draws high current at start so induction motor starters like star-delta, autotransformer etc. used to start the synchronous motor as an induction motor.

3. As a Slip Ring Induction Motor

- The method of starting synchronous motor as a squirrel cage induction motor does not provide high starting torque.
- So to achieve this, instead of shorting the damper winding, it is designed to form a three phase star or delta connected winding. The three ends of this winding are brought out through slip rings.
- An external rheostat then can be introduced in series with the rotor circuit.
- So when stator is excited, the motor starts as a slip ring induction motor and due to resistance added in the rotor provides high starting torque.



- The resistance is then gradually cut-off, as motor gathers speed.

- When motor attains speed near synchronous, d.c. excitation is provided to the rotor, then motor gets pulled into synchronism and starts rotating at synchronous speed.
- The damper winding is shorted by shorting the slip rings. The initial resistance added in the rotor not only provides high starting torque but also limits high inrush of starting current. Hence it acts as a rotor resistance starter.

4. Using Small D.C. Machine

- Large synchronous motors are provided with a coupled d.c. machine.
- This machine is used as a d.c. motor to rotate the synchronous motor at a synchronous speed.
- Then the excitation to the rotor is provided.
- Once motor starts running as a synchronous motor, the same d.c. machine acts as a d.c. generator called exciter.
- The field of the synchronous motor is then excited by this exciter itself.

Behaviour of Synchronous Motor on Loading

- When synchronous motor rotates at synchronous speed, the stationary stator (armature) conductors cut the flux produced by rotor.
- Due to this there is an induced e.m.f. in the stator which according to Lenz's law opposes the supply voltage.
- This induced e.m.f. is called back e.m.f. in case of synchronous motor.
- It is denoted as E_{bph} i.e. back e.m.f. per phase.
- This gets generated as the principle of alternator and hence alternating in nature and its magnitude can be calculated by the equation,

$$E_{bph} = 4.44\ K_c\ K_d\ \Phi\ f\ T_{ph}$$

or $$E_{bph}\ \alpha\ \Phi$$

- As speed is always synchronous, the frequency is constant and hence magnitude of such back e.m.f. can be controlled by changing the flux (p produced by the rotor.
- Voltage Equation of Synchronous motor is given by

$$V_{ph} = E_{bph} + I_{aph}\ Z_s$$

Where $Z_s = R_a + j\ X_s$ Ω/phase

V_{ph} = supply voltage/Phase

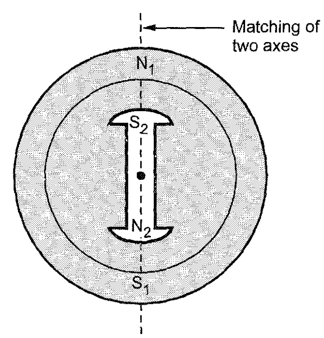
Therefore, $I_{aph} = \frac{V_{ph} - E_{bph}}{I_{aph}}$

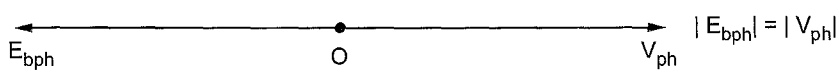
Ideal Condition on No Load

- The ideal condition on no load can be assumed by neglecting various losses in the motor.

$$V_{ph} = E_{bph}$$

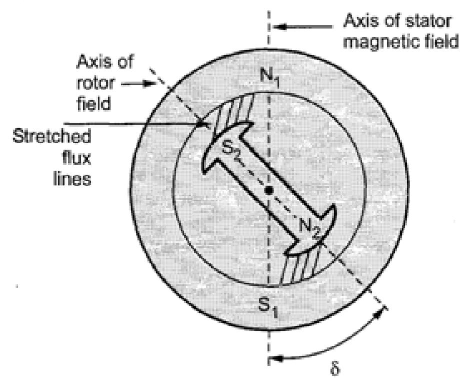
- Under this condition, the magnetic locking between stator and rotor is in such a way that the magnetic axes of both, coincide with each other as shown in the Fig.
- As this is possible only under no losses condition, is said to be ideal in case of synchronous motor.
- As magnitude of E_{bPh} and V_{ph} is same and E_{bph} opposes V_{ph}, the phasor diagram for this condition can be shown as in the Fig.





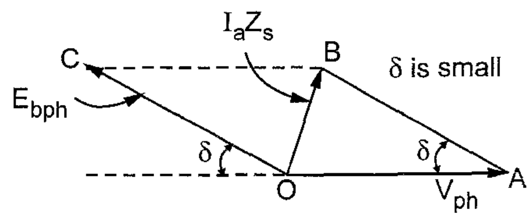
- But, $I_{aph} = \frac{V_{ph} - E_{bph}}{I_{aph}}$
- But the vector difference V_{ph} — E_{bph} = 0 as seen from the phasor diagram. Hence I_a = 0 under no losses condition.
- In practice this is impossible. Motor has to supply mechanical losses and iron losses along with small copper losses. To produce torque to overcome these losses, motor draws a current from the supply.

Synchronous Motor on No Load (With Losses)



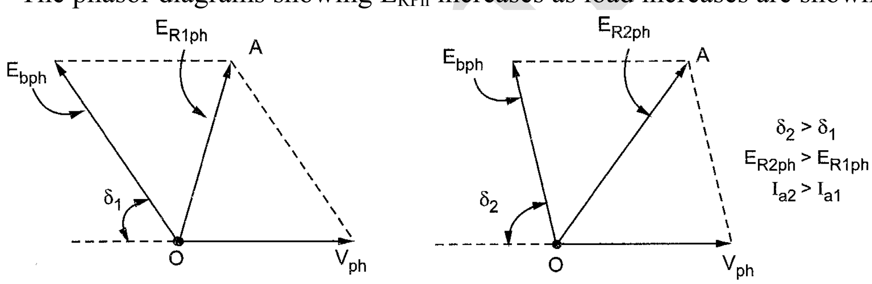
- Due to the various losses practically present on no load, the magnetic locking exists between stator and rotor but in such a way that there exists a small angle difference between the axes of two magnetic fields as shown in the Fig.
- So the rotor axis falls back with respect to stator axis by angle 'δ' as shown in Fig.
- This angle decides the amount of current required to produce the torque to supply various losses.
- Hence this angle is called load angle, power angle, coupling angle, torque angle or angle of retardation and denoted as δ.
- The Phasor diagram for no load condition with losses is shown in fig.
- The resultant phasor is denoted as E_{RPh} which is the product of armature current per phase and armature impedance per phase.

  $E_{Rph} = I_{aph} Z_s$
- This resultant decides the amount of current I_{aph} to be drawn to produce the torque which meets the various losses present in the synchronous motor.
- Under no load condition, δ is very small and hence E_{RPh} is also very small.



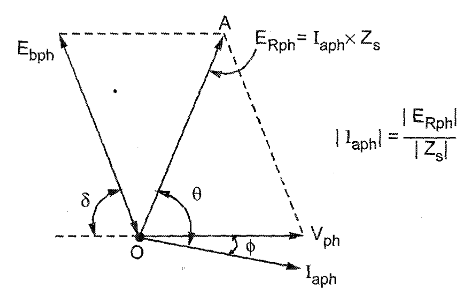
Synchronous Motor on Load

- As the load on the synchronous motor increases, there is no change in its speed. But the load angle δ i.e. the angle by which rotor axis retards with respect to stator axis changes.
- Hence as load increases, δ increases but speed remains synchronous.
- As δ increases, though E_{bph} and V_{ph} magnitudes are same, displacement of E_{bPh} from its ideal position increases. Hence the vector difference V_{ph} — E_{bph} increases.
- As synchronous impedance is constant, the magnitude of I_{aph} drawn by the motor increases as load increases. This current produces the necessary torque which satisfies the increased load demand. The magnetic locking still exists between the rotor and stator.
- The phasor diagrams showing E_{RPh} increases as load increases are shown in Fig. (a) and (b).



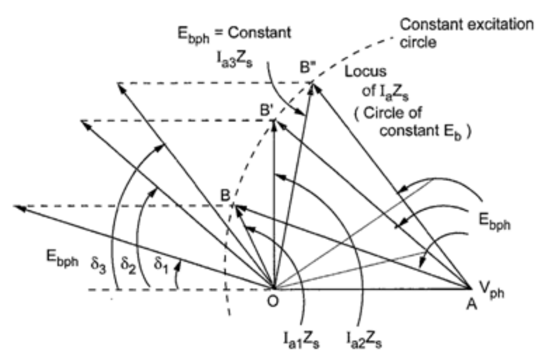
(a) Light load

(b) Heavy load



Constant Excitation Circle

- As E_{bph} depends on flux, for constant excitation E_{bph} is constant.
- For constant excitation, if load is varied then δ keeps on changing, due to which V_{ph} - E_{bph} = E_{Rph} = $I_{aph}Z_s$ keeps on changing.
- The locus of extremities of E_{Rph} = $I_{aph}Z_s$ is a circle and as Z_s is constant, represents current locus for the synchronous motor under constant excitation and variable load conditions.
- As δ increases, I_{aph} Z_s increases and motor draws more current.
- As load decreases, δ decreases hence I_{aph} Z_s decreases and motor draws less current. Such a current locus is shown in the Fig.



- The mechanical power developed by the synchronous motor is given by

$$P = \frac{E_b . V_{ph}}{Z_s} \cos(\theta - \delta) - \frac{{E_b}^2}{Z_s} \cos\theta$$

Where E_b = Induced e.m.f. which is constant for constant excitation

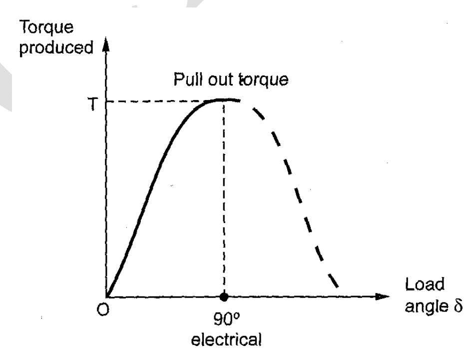
θ = Angle decided by $Z_s = \tan^{-1} \frac{X_a}{R_a}$

Now neglecting R_a, $\theta = 90°$ and using in equation (1),

$$P_m = \frac{E_b V_{ph}}{Z_s} \cos(90 - \delta) = \frac{E_b V_{ph}}{X_s} \sin\delta, \quad |Z_s| = |X_s|$$

$$T = \frac{P_m}{\omega_s} = \frac{\left(\frac{E_b V_{ph}}{X_s} \sin\delta\right)}{\left(\frac{2\pi N_s}{60}\right)} \quad \text{i.e.} \quad \boxed{T \propto \sin\delta}$$

- As angle δ increases, the magnetic flux lines producing the force of attraction between the two get more and more stretched. This weakens the force maintaining the magnetic locking, though torque produced by the motor increases.
- As δreaches upto 90^0 electrical i.e. half a pole pitch, the stretched flux lines get broken and hence magnetic locking between the stator and rotor no longer exists. The motor comes out of synchronism.
- So torque produced at δ equal to 90° electrical is the maximum torque, a synchronous motor can produce, maintaining magnetic locking i.e. synchronism. Such a torque is called pull out torque.
- The relationship between torque produced and load angle δ is shown in the Fig.



Synchronous Motor Connected to Infinite Bus Bar

- The synchronous motor connected to an infinite bus bar behaves similarly for the changes in the load at constant excitation.
- As the load increases, the load angle increases, current increases and power factor changes.
- The changes in the power factor depends on the excitation used for synchronous motor i.e. whether it is normally excited ($E_{bph} = V_{ph}$), over excited ($E_{bph} > V_{ph}$) or under excited ($E_{bph} < V_{ph}$).
- Thus effect of change in load on synchronous motor connected to an infinite bus bar can be summarized as,
  1. Irrespective of excitation, as load increases, the load angle δ and armature current I_a increases.
  2. When the motor is normally excited (E_{bph} = Vph), then as load increases, the change in current is more significant than the change in power factor. The power factor tends to become more and more lagging as the load increases.
  3. When the motor is over excited or under excited, the power factor changes are more significant than the changes in the current as load changes.
  4. 4. When the motor is over excited or under excited, the power factor tends to approach to unity as the load increases.

Torques in Synchronous Motor

1) Starting torque: This is the torque developed by the synchronous motor at start when rated voltage is applied to the stator. It is also called breakaway torque. It is necessary to overcome friction and inertia.

2) Running torque: It is the torque developed by the motor under running conditions. It is decided by the output rating of the motor and speed of the driven machine.

3) Pull in torque: Initially synchronous motor is rotated at a speed slightly less than the synchronous speed. When speed is near to synchronous, excitation is switched on and motor gets pulled into synchronism and starts rotating at the synchronous speed. The amount of torque developed by the motor at the time of pulling into synchronism is called pull in torque.

4) Pull out torque: When the synchronous motor is loaded, the rotor falls back with respect to stator by an angle called load angle δ. As δ increases, magnetic locking between stator and rotor decreases. At $\delta = 90^0$, the torque developed is maximum by the motor and magnetic locking is very weak. Any further increase in the load pulls motor out of synchronism and motor stops. Thus the maximum torque developed by the synchronous motor without pulling out of synchronism is called pull out torque.

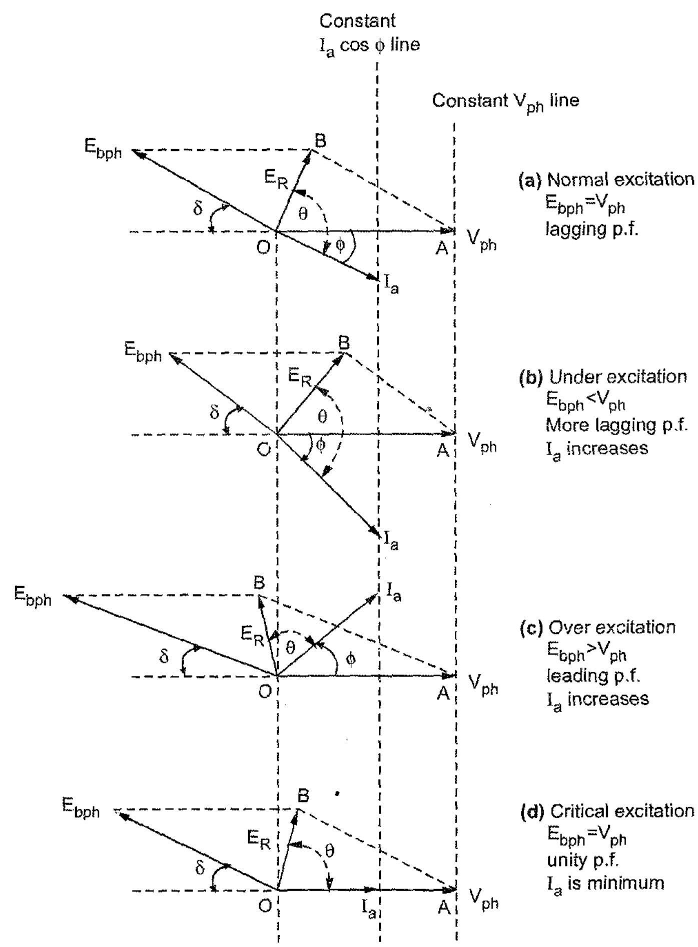
OPERATION OF SYNCHRONOUS MOTOR AT CONSTANT LOAD VARIABLE EXCITATION

- If excitation i.e. field current is changed keeping load constant, the synchronous motor reacts by changing its power factor of operation.
- Consider a synchronous motor operating at a certain load. The corresponding load angle is δ.

Normal Excitation

- $E_b = V$ i.e. induced e.m.f. is equal to applied voltage.

Motor is drawing certain current I_a from the supply and power input to the motor is P_{in}. The power factor of the motor is lagging in nature as shown in the Fig. (a).



Now when excitation is changed, E_b changes but there is hardly any change in the losses of the motor. So the power input also remains same for constant load demanding same power output.

$$P_{in} = \sqrt{3}\, V_L I_L \cos\Phi = 3\, V_{ph} I_{aph} \cos\Phi$$

Most of the times, the voltage applied to the motor is constant. Hence for constant power input as V_{ph} is constant, $I_{aph} \cos\Phi$ remains constant.

So motor adjusts its $\cos\Phi$ i.e. p.f. nature and value so that $I_a \cos\Phi$ remains constant when excitation of the motor is changed keeping load constant. This is the reason why synchronous motor reacts by changing its power factor to variable excitation conditions.

Under Excitation

- When the excitation is adjusted in such a way that the magnitude of induced e.m.f. is less than the applied voltage ($E_b < V$) the excitation is called Under excitation.
- Due to this, E_R increases in magnitude. This means for constant Z_s, current drawn by the motor increases.
- But E_R phase shifts in such a way that, phasor I_a also shifts (as $E_R \wedge I_a = \theta$) to keep $I_a \cos\Phi$ component constant. This is shown in the Fig. (b).
- So in under excited condition, current drawn by the motor increases. The p.f. $\cos\Phi$ decreases and becomes more and more lagging in nature.

Over Excitation

- The excitation to the field winding for which the induced e.m.f. becomes greater than applied voltage ($E_b > V$), is called over excitation.
- Due to increased magnitude of E_b, E_R also increases in magnitude.
- But the phase of E_R also changes. Now $E_R \wedge I_a = \theta$ is constant, hence I_a also changes its phase.
- So Φ changes. The I_a increases to keep $I_a \cos\Phi$ constant as shown in Fig. (c).
- The phase of E_R changes so that I_a becomes leading with respect to V_{ph} in over excited condition.

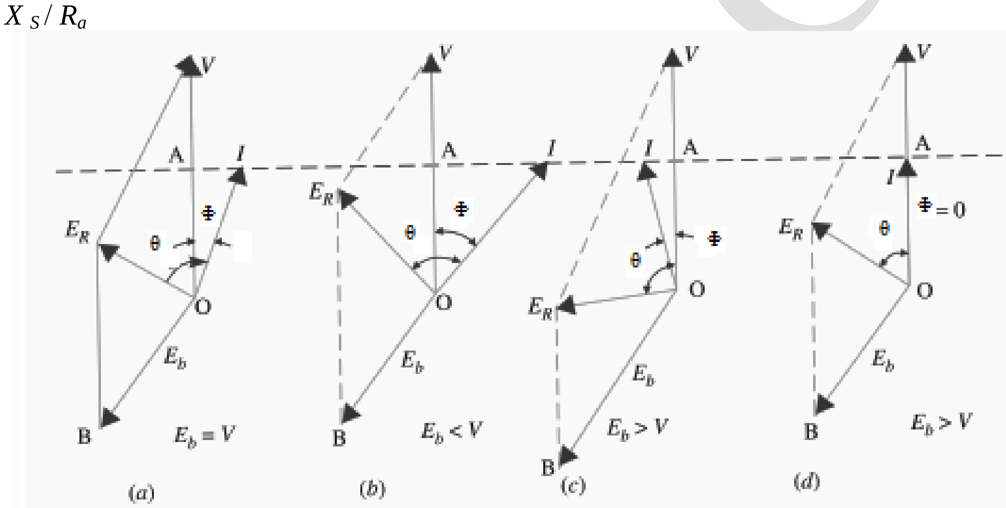
- So power factor of the motor becomes leading in nature.
- So overexcited synchronous motor works on leading power factor. So power factor decreases as over excitation increases but it becomes more and more leading in nature.

Critical Excitation

- When the excitation is changed, the power factor changes. The excitation for which the power factor of the motor is unity (cos Φ = 1) is called critical excitation.
- Then I_{aph} is in phase with V_{ph}.
- Now I_a cos Φ must be constant, cos Φ = 1 is at its maximum hence motor has to draw minimum current from supply for unity power factor condition.
- So for critical excitation, cos Φ = 1 and current drawn by the motor is minimum compared to current drawn by the motor for various excitation conditions. This is shown in the Fig. (d).

V-Curves and Inverted V-Curves

- The value of excitation for which back e.m.f. E_b is equal (in magnitude) to applied voltage V is known as 100% excitation.
- Consider a synchronous motor in which the mechanical load is constant (and hence output is also constant if losses are neglected). Fig. (*a*) shows the case for 100% excitation *i.e.*, when $E_b = V$.
- The armature current *I* lags behind V by a small angle Φ . Its angle θ with E_R is fixed by stator constants *i.e.*
  $\tan\theta = X_S / R_a$



- In Fig. (*b*) excitation is less than 100% *i.e.*, $E_b < V$. Here, E_R is advanced clockwise and so is armature current (because it lags behind E_R by fixed angle θ).
- The magnitude of *I* is increased but its power factor is decreased (Φ has increased).
- Because input as well as V are constant, hence the power component of *I i.e.*, *I* cos Φ remains the same as before, but wattless component *I* sin Φ is increased.
- Hence, as excitation is decreased, *I* will increase but p.f. will decrease so that power component of *I i.e.*, *I* cos Φ = *OA* will remain constant.
- In fact, the locus of the extremity of current vector would be a straight horizontal line as shown.
- Fig. (*c*) represents the condition for overexcited motor *i.e.* when $E_b > V$.
- Here, the resultant voltage vector E_R is pulled anticlockwise and so is *I*.
- It is seen that now motor is drawing a leading current.
- It may also happen for some value of excitation, that *I* may be in phase with V *i.e.*, p.f. is unity [Fig. (*d*)]. At that time, the current drawn by the motor would be ***minimum***.

Two important points stand out clearly from the above discussion:

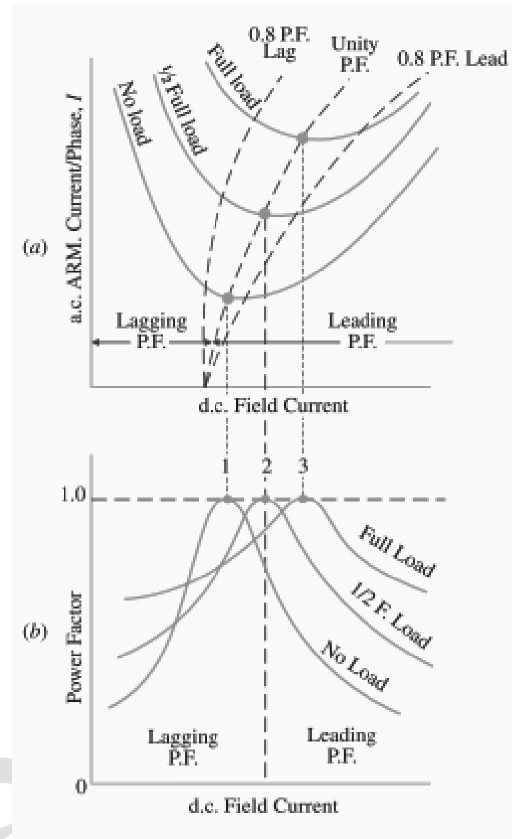
(i) The magnitude of armature current varies with excitation. The current has large value both for low and high values of excitation (though it is lagging for low excitation and leading for higher excitation). In between, it has minimum value corresponding to a certain excitation.

The variations of *I* with excitation are shown in Fig. (*a*) which are known as '*V*' curves because of their shape.

(ii) For the same input, armature current varies over a wide range and so causes the power factor also to vary accordingly.

When over-excited, motor runs with leading p.f. and with lagging p.f. when under-excited. In between, the p.f. is unity.

The variations of *p.f.* with excitation are shown in Fig. (*b*). The curve for p.f. looks like inverted '*V*' curve. It would be noted that ***minimum armature current corresponds to unity power factor***.



EXPRESSION FOR BACK E.M.F. OR INDUCED E.M.F. PER PHASE IN SYNCHRONOUS MOTOR (E_{bph})

Case i] Under excitation, $E_{bph} < V_{ph}$.

$$Z_s = R_a + j\,X_s = |\,Z_s\,| \angle\ \theta\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{X_s}{R_a}\right)$$

$$E_{Rph} \hat{\ } I_{aph} = \theta,\ I_a \text{ lags } E_R \text{ always by angle } \theta.$$

$$V_{ph} = \text{Phase voltage applied}$$

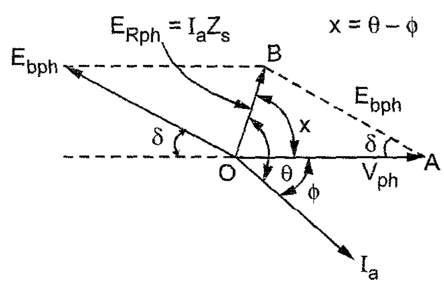
$$E_{bph} = \text{Back e.m.f. induced per phase}$$

$$E_{Rph} = I_a \times Z_s \text{ V} \qquad \text{... Per phase}$$

Let p.f. be cos ϕ, lagging as under excited,

$$V_{ph} \hat{\ } I_{aph} = \phi$$

Phasor diagram is shown in the Fig.



Applying cosine rule to Δ OAB,

$$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2\,V_{ph}\,E_{Rph} \times \cos(V_{ph} \hat{\ } E_{Rph})$$

but $V_{ph} \wedge E_{Rph} = x = \theta - \phi$

$$\therefore \quad (E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2 V_{ph} E_{Rph} \times \cos(\theta - \phi)$$

where $E_{Rph} = I_{aph} \times Z_s$ ——— 1

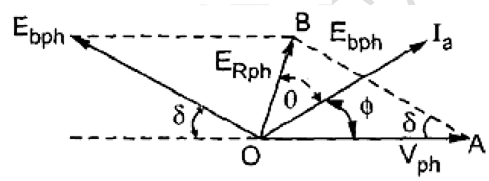
Applying sine rule to Δ OAB,

$$\frac{E_{bph}}{\sin x} = \frac{E_{Rph}}{\sin\delta} \quad \text{i.e.} \quad \sin\delta = \frac{E_{Rph}\sin(\theta-\phi)}{E_{bph}} \qquad \text{——— 2}$$

So once E_{bph} is calculated, load angle δ can be determined by using sine rule.

Case ii] Over excitation, $E_{bph} > V_{ph}$

p.f. is leading in nature.



$$E_{Rph} \hat{} I_{aph} = \theta$$

$$V_{ph} \hat{} I_{aph} = \phi$$

The phasor diagram is shown in the Fig.

Applying cosine rule to Δ OAB,

$$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2 V_{ph} E_{Rph} \times \cos(V_{ph} \wedge E_{Rph})$$

$$V_{ph} \wedge E_{Rph} = \theta + \phi$$

$$\therefore \quad (E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2 V_{ph} E_{Rph} \cos(\theta + \phi) \qquad \text{——— 3}$$

But θ + φ is generally greater than 90°

∴ cos (θ + φ) becomes negative, hence for leading p.f., $E_{bph} > V_{ph}$.

Applying sine rule to Δ OAB,

$$\frac{E_{bph}}{\sin(E_{Rph} \wedge V_{ph})} = \frac{E_{Rph}}{\sin\delta}$$

$$\therefore \quad \sin\delta = \frac{E_{Rph}\sin(\theta+\phi)}{E_{bph}} \qquad \text{——— 4}$$

Hence load angle δ, can be calculated once E_{bph} is known.

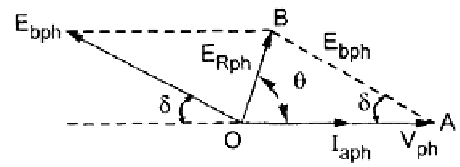
Case iii] Critical excitation

In this case $E_{bph} \cong V_{ph}$, but p.f. of synchronous motor is unity.

$\therefore \quad \cos\phi = 1 \quad \therefore \quad \phi = 0°$

i.e. V_{ph} and I_{aph} are in phase.

and $E_{Rph} \wedge I_{aph} = \theta$



Phasor diagram is shown in the Fig.

Applying cosine rule to Δ OAB,

$$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2\,V_{ph}\,E_{Rph}\cos\theta \qquad \text{——— } 5$$

Applying sine rule to Δ OAB,

$$\frac{E_{bph}}{\sin\theta} = \frac{E_{Rph}}{\sin\delta}$$

$$\therefore \quad \boxed{\sin\delta = \frac{E_{Rph}\sin\theta}{E_{bph}}} \qquad \text{——— } 6$$

Thus in general the induced e.m.f. can be obtained by,

$$(E_{bph})^2 = (V_{ph})^2 + (E_{Rph})^2 - 2\,V_{ph}\,E_{Rph}\cos(\theta \pm \phi)$$

+ sign for leading p.f. while – sign for lagging p.f.

POWER FLOW IN SYNCHRONOUS MOTOR

Net input to the synchronous motor is the three phase input to the stator.

Therefore, $P_{in} = \sqrt{3}\,V_L I_L \cos\Phi$ W

where V_L = Applied line voltage

I_L = Line current drawn by the motor

Cos Φ = Operating p.f. of synchronous motor

$P_{in} = 3\,V_{ph} I_{ph} \cos\Phi$ W

Now in stator, due to its resistance Ra per phase there are stator copper losses.

Total stator copper losses = $3 \times (I_{aph})^2 \times R_a$ W

∴ The remaining power is converted to the mechanical power, called **gross mechanical power developed** by the motor denoted as P_m.

$$\therefore \quad \boxed{P_m = P_{in} - \text{Stator copper losses}}$$

Now $\quad P = T \times \omega$

$$\therefore \quad P_m = T_g \times \frac{2\pi N_s}{60} \qquad \text{as speed is always } N_s$$

$$\therefore \quad \boxed{T_g = \frac{P_m \times 60}{2\pi N_s}\ \text{Nm}}$$

This is the **gross mechanical torque developed.** In d.c. motor, electrical equivalent of gross mechanical power developed is $E_b \times I_a$, similar in synchronous motor the electrical equivalent of gross mechanical power developed is given by,

$$P_m = 3\,E_{bph} \times I_{aph} \times \cos(E_{bph} \,\hat{}\, I_{aph})$$

i) For lagging p.f., $\boxed{E_{bph} \,\hat{}\, I_{aph} = \phi - \delta}$

ii) For lagging p.f., $\boxed{E_{bph} \,\hat{}\, I_{aph} = \phi + \delta}$

iii) For unity p.f., $\boxed{E_{bph} \,\hat{}\, I_{aph} = \delta}$

In general,

$P_m = 3\ E_{bph}\ I_{aph} \cos(\Phi \pm \delta)$

Positive sign for leading p.f. and Negative sign for lagging p.f.

Net output of the motor then can be obtained by subtracting friction and windage i.e. mechanical losses from gross mechanical power developed.

$$\therefore \quad \boxed{P_{out} = P_m - \text{mechanical losses.}}$$

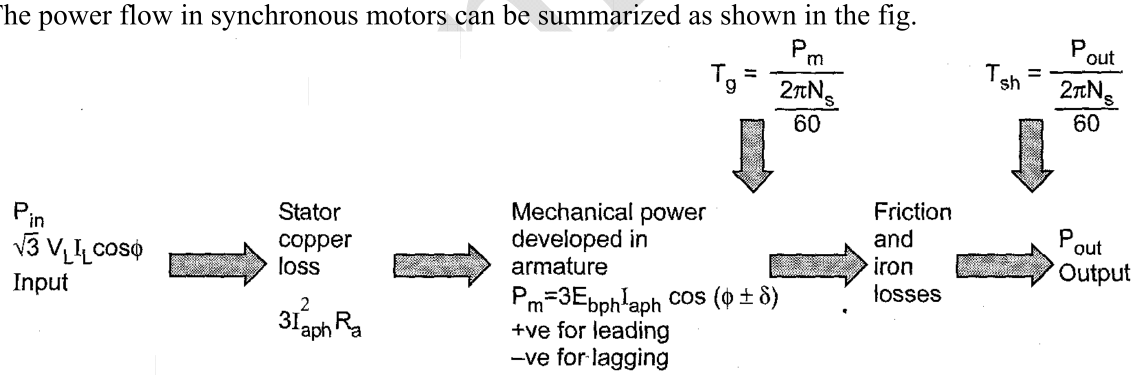
$$\therefore \quad T_{shaft} = \frac{P_{out} \times 60}{2\pi N_s} \text{ Nm}$$

where T_{shaft} = Shaft torque available to load

P_{out} = Power available to load

$$\therefore \quad \boxed{\%\,\eta = \frac{P_{out}}{P_{in}} \times 100} \quad \text{... Overall efficiency}$$

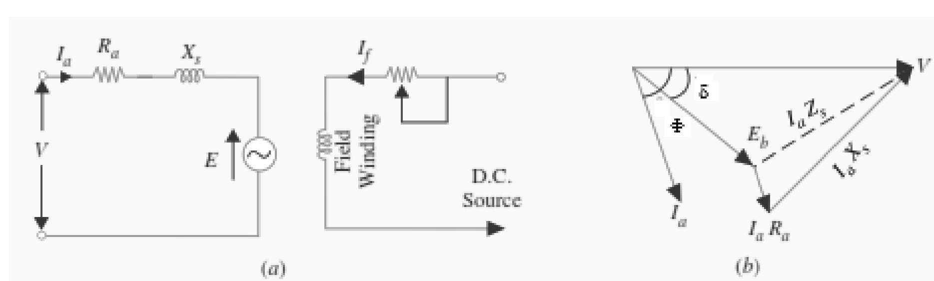
The power flow in synchronous motors can be summarized as shown in the fig.



Equivalent Circuit of a Synchronous Motor

Fig. (*a*) Shows the equivalent circuit model for one armature phase of a cylindrical rotor synchronous motor.

It is seen from Fig. (*b*) that the phase applied voltage *V* is the vector sum of reversed back e.m.f. *i.e.*, $-E_b$ and the impedance drop $I_a\ Z_S$.



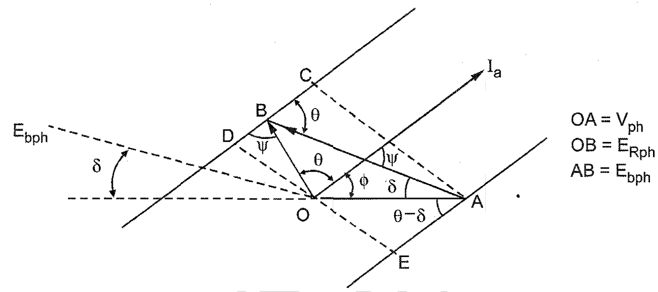
(*a*) (*b*)

In other words, $V = (-E_b + I_a Z_S)$.
The angle δ between the phasor for V and E_b is called the load angle or power angle of the synchronous motor.

Expression for Power Developed by a Synchronous Motor

Consider the phasor diagram of a synchronous motor running on leading power factor cos Φ as shown in the Fig.

The line CD is drawn at an angle θ to AB.
The lines AC and DE are perpendicular to CD and AE.



$OB = E_{Rph} = I_a Z_s$ and $\angle OBD = \psi$

and angle between $AB = E_{bph}$ and I_{aph} is also ψ.

The mechanical per phase power developed is given by,

$$P_m = E_{bph} I_{aph} \cos(E_{bph} \wedge I_{aph}) \qquad \text{------------- (1)}$$

$$= E_{bph} I_{aph} \cos(\psi)$$

In triangle OBD,

$$BD = OB \cos\psi = I_a Z_s \cos\psi$$

$$OD = OB \sin\psi = I_a Z_s \sin\psi$$

Now $\quad BD = CD - BC = AE - BC$

$$AE = OA \cos(\theta - \delta) = V_{ph} \cos(\theta - \delta) \qquad \text{--------------- (2)}$$

Substituting in equation **2**

$$\therefore \quad I_a Z_s \cos\psi = V_{ph} \cos(\theta - \delta) - E_b \cos\theta$$

$$\therefore \quad I_a \cos\psi = \frac{V_{ph}}{Z_s} \cos(\theta - \delta) - \frac{E_b}{Z_s} \cos\theta \qquad \text{------------- (3)}$$

All values are per phase values

Substituting equation (3) in equation (1),

$$\therefore \quad P_m = E_b \left[\frac{V_{ph}}{Z_s} \cos(\theta - \delta) - \frac{E_b}{Z_s} \cos\theta \right]$$

$$\therefore \quad \boxed{P_m = \frac{E_b V_{ph}}{Z_s} \cos(\theta - \delta) - \frac{E_b^2}{Z_s} \cos\theta} \qquad \text{--------------- (4)}$$

This is the expression for the mechanical power developed interms of the load angle δ and the internal machine angle Φ, for constant voltage V_{ph} and constant E_b i.e. excitation.

$$\text{Gross torque } T_g = \frac{P_m}{\omega} = \frac{P_m}{\left(\frac{2\pi N_s}{60}\right)}$$

where N_s = Synchronous speed in r.p.m

$$T_g = \frac{60}{2\pi}\frac{P_m}{N_s} = \frac{9.55\,P_m}{N_s} \qquad \text{-------------- (5)}$$

Condition for Maximum Power Developed

The value of δ for which the mechanical power developed is maximum can be obtained as,

$$\frac{dP_m}{d\delta} = 0$$

$$\therefore \quad \frac{d}{d\delta}\left[\frac{E_b V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{E_b^2}{Z_s}\cos\theta\right] = 0$$

$$\therefore \quad \frac{E_b V_{ph}}{Z_s}\,.\,\sin(\theta-\delta)\,(-1) = 0 \quad \text{i.e.} \quad \sin(\theta-\delta) = 0$$

$$\therefore \quad \boxed{\theta = \delta} \qquad \text{----------- (6)}$$

The Value of Maximum Power Developed

The value of maximum power developed can be obtained by substituting θ = δ in the equation of P_m.

$$(P_m)_{max} = \frac{E_b V_{ph}}{Z_s}\cos(0) - \frac{E_b^2}{Z_s}\cos(\delta)$$

$$\therefore \quad (P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2}{Z_s}\cos\delta \qquad \text{------------ (7)}$$

$$\therefore \quad (P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2}{Z_s}\cos\theta \qquad \text{------------- (8)}$$

When R_a is negligible, $\theta = 90°$ and $\cos(\theta) = 0$ hence,

$$\therefore \quad \boxed{(P_m)_{max} = \frac{E_b V_{ph}}{Z_s}} \qquad \text{... when } R_a \text{ is negligible}$$

We know that

$$Z_s = R_a + j\,X_s = |Z_s| \angle \theta$$

$$\therefore \quad R_a = Z_s\cos\theta \text{ and } X_s = Z_s\sin\theta$$

Substituting $\cos\theta = R_a/Z_s$ in equation (8) we get,

$$\therefore \quad \boxed{(P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2 R_a}{Z_s^2}} \qquad \text{----------- (9)}$$

The torque is given by P/ω hence the maximum torque developed by a synchronous motor is,

$$T_{max} = \frac{(P_m)_{max}}{\omega_s} = \frac{(P_m)_{max}}{\left(\frac{2\pi N_s}{60}\right)} \quad \text{Nm}$$

$$\therefore \quad \boxed{T_{max} = \frac{\left[\frac{E_b V_{ph}}{Z_s} - \frac{E_b^2 R_a}{Z_s^2}\right]}{\left(\frac{2\pi N_s}{60}\right)}}$$

Condition for Excitation When Motor Develops $(P_m)_{max}$

The excitation controls E_b. Hence the condition of excitation can be obtained as,

$$\frac{dP_m}{dE_b} = 0$$

$$\therefore \quad \frac{d}{dE_b}\left[\frac{E_b V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{E_b^2}{Z_s}\cos\theta\right] = 0$$

Assume load constant hence δ constant.

$$\therefore \quad \frac{V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{2E_b}{Z_s}\cos\theta = 0$$

but $\theta = \delta$ for $P_m = (P_m)_{max}$

$$\therefore \quad \frac{V_{ph}}{Z_s} - \frac{2E_b}{Z_s}\cos\theta = 0$$

Substitute $\cos\theta = \frac{R_a}{Z_s}$

$$\therefore \quad \frac{V_{ph}}{Z_s} - \frac{2E_b}{Z_s}\cdot\frac{R_a}{Z_s} = 0$$

$$\therefore \quad \boxed{E_b = \frac{V_{ph} Z_s}{2R_a}}$$

This is the required condition of excitation.

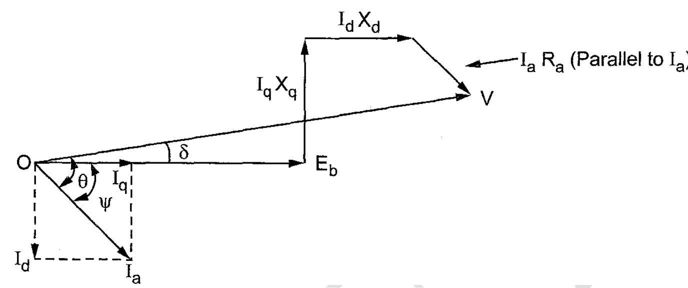
The corresponding value of maximum power is,

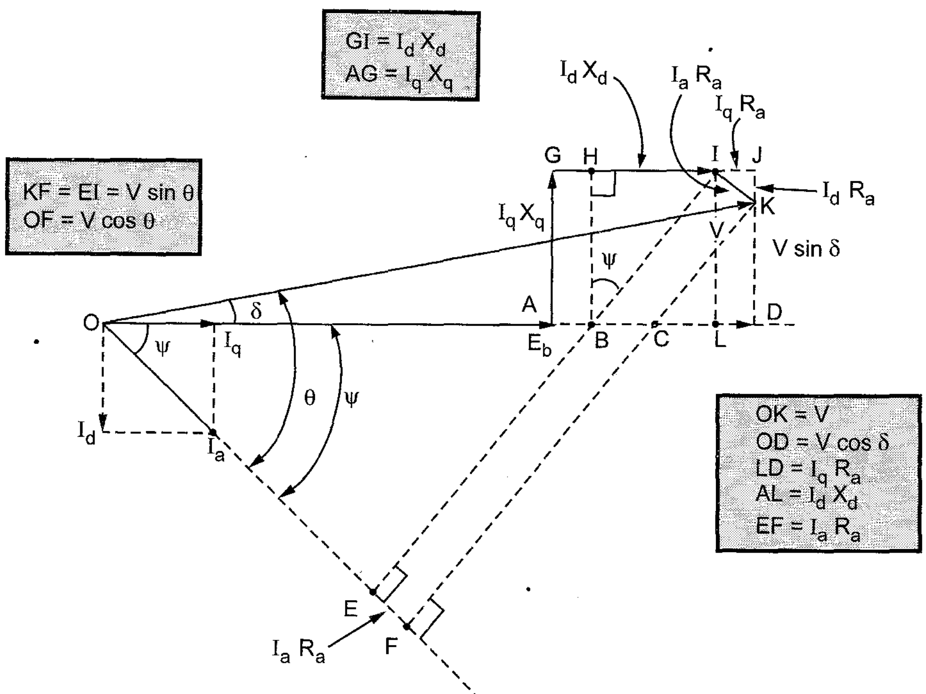
$$\therefore \quad \boxed{(P_m)_{max} = \frac{V_{ph}^2}{2R_a} - \frac{V_{ph}^2}{4R_a}}$$

SALIENT POLE SYNCHRONOUS MOTOR (Two Reaction Theory for Synchronous Motor)

The two reactances according to this theory are direct axis reactance X_d and quadrature axis reactance X_q. The motor armature current has two components I_d and Iq. The phasor diagram for the synchronous motor with a lagging power factor is shown in figure.

The phasor I_qX_q is such that I_q lags I_qX_q by 90^0 while the phasor I_dX_d is such that I_d lags I_dX_d by 90°. The angle between V and E_b is δ, V and I_a is θ and Eb and I_a is ψ. The phasor diagram can be further modified as shown in the Fig.





From the phasor diagram,

$$\boxed{E_b = OD - DL - AL = V \cos \delta - I_q R_a - I_d X_d}$$

$$AG = I_q X_q = DK + KJ = V \sin\delta + I_d R_a$$

In Δ BHI, ∠ HBI = ψ

$$\therefore \quad \cos \psi = \frac{BH}{BI} = \frac{AG}{BI} = \frac{I_q X_q}{BI}$$

From the triangle of I_a, I_d and I_q,

$$\therefore \quad \cos \psi = \frac{I_q}{I_a}$$

$$\therefore \quad \frac{I_q}{I_a} = \frac{I_q X_q}{BI} \qquad \text{i.e.} \qquad BI = I_a X_q$$

In Δ OBE,

$$\tan\psi = \frac{BE}{OE} = \frac{EI - BI}{OF - EF} = \frac{V\sin\theta - I_a X_q}{V\cos\theta - I_a R_a}$$

$$\therefore \quad \boxed{\tan\psi = \frac{V\sin\theta - I_a X_q}{V\cos\theta - I_a R_a}} \text{ (Lagging p.f.)}$$

$$\therefore \quad \boxed{\begin{aligned} \delta &= \theta - \psi \quad \text{for lagging p.f.} \\ \delta &= \psi - \theta \quad \text{for leading p.f.} \end{aligned}}$$

For the leading power factor,

$$\therefore \quad \boxed{\begin{aligned} E_b &= V\cos\delta + I_q R_a - I_d X_d \\ \tan\psi &= \frac{V\sin\theta - I_a X_q}{V\cos\theta + I_a R_a} \end{aligned}}$$

The expression for the total power developed by a salient pole synchronous motor is given by,

$$\therefore \quad \boxed{P_m = 3\times\left\{\frac{E_b V}{X_d}\sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta\right\} \text{ W}}$$

For **maximum power** developed, load angle δ can be obtained as ,

$$\frac{dP_m}{d\delta} = 0 \quad \text{i.e.} \quad \frac{d}{d\delta}\left[\frac{E_b V}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] = 0$$

$$\therefore \quad \frac{E_b V}{X_d}\cos\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)2\cos 2\delta = 0$$

$$\therefore \quad \frac{E_b V}{X_d}\cos\delta + V^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\cos 2\delta = 0$$

The expression of dPm / dδ gives the rate of change of power with respect to load angle δ and is called stability factor, rigidity factor or the stiffness of coupling.

Blondel Diagram [Constant Power Circle]

For a synchronous motor, the power input to the motor per phase is given by,

$$P_{in} = V_{ph} I_{aph} \cos\phi \qquad \text{... Per phase}$$

The gross mechanical power developed per phase will be equal to the difference between P_{in} per phase and the per phase copper losses of the winding.

Copper loss per phase $= (I_{aph})^2 R_a$

$$\therefore \quad P_m = V_{ph} I_{aph} \cos\phi - (I_{aph})^2 R_a \qquad \text{... Per phase}$$

For mathematical convenience let $\quad V_{ph} = V$ and $I_{aph} = I$,

$$\therefore \quad P_m = VI\cos\phi - I^2 R_a$$

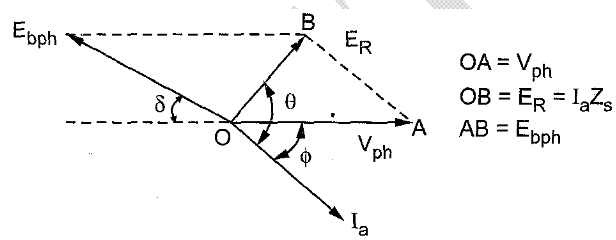
$$\therefore \quad I^2 R_a - V I \cos\phi + P_m = 0$$

$$\therefore \quad I^2 - \frac{V I \cos\phi}{R_a} + \frac{P_m}{R_a} = 0 \qquad \text{---------- (1)}$$

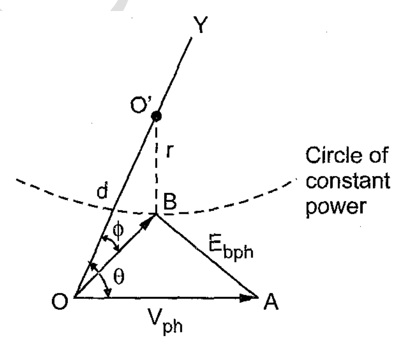
The equation (1) represents polar equation to a circle. To obtain this circle in a phasor diagram, draw a line OY at an angle θ with respect to OA.

∟YOA = θ

∟YOB = Φ



The circle represented by equation (1) has a centre at some point 0' on the line OY. The circle drawn with centre as 0' and radius as O'B represents circle of constant power. This is called Blondel diagram, shown in the Fig.



Thus if excitation is varied while the power is kept constant, then working point B will move along the circle of constant power.

Let O'B = Radius of circle = r

OO' = Distant d

Applying cosine rule to triangle OBO',

$r^2 = (OB)^2 + (OO')^2 - 2\,(OB)\,(OO')\cos(OB \wedge OO')$ ----- (2)

Now OB represents resultant E_R which is I_aZ_s. Thus OB is proportional to current and when referred to OY represents the current in both magnitude and phase.

OB = I_a = I say

Substituting various values in equation (2) we get,

$$r^2 = I^2 + d^2 - 2dI\cos\phi$$

$$\therefore \quad I^2 - 2d\,I\cos\phi + (d^2 - r^2) = 0 \qquad \text{------- (3)}$$

Comparing equations (1) and (3) we get,

OO' = d = $V/2R_a$ ------ (4)

Thus the point O' is independent of power P_m and is a constant for a given motor operating at a fixed applied voltage V. Comparing last term of equations (1) and (3),

$$d^2 - r^2 = \frac{P_m}{R_a}$$

$$\therefore \quad r^2 = \left(d^2 - \frac{P_m}{R_a}\right)$$

$$\therefore \quad r = \sqrt{d^2 - \frac{P_m}{R_a}} = \sqrt{\left(\frac{V}{2R_a}\right)^2 - \frac{P_m}{R_a}}$$

$$\therefore \quad \boxed{r = \frac{1}{2R_a}\sqrt{V^2 - 4P_m R_a}} \qquad \text{--------- (5)}$$

The equation shows that as power P_m must be real, then $4\,P_mR_a \geq V^2$. The maximum possible power per phase is,

$$4\,(P_m)_{max}\,R_a = V^2$$

$$\therefore \quad \boxed{(P_m)_{max} = \frac{V^2}{4\,R_a}} \qquad \text{------------ (6)}$$

And the radius of the circle for maximum power is zero. Thus at the time of maximum power, the circle becomes a point O'.

While when the power $P_m = 0$, then

$r = V / (2\,R_a)$

This shows that the circle of zero power passes through the points O and A.

The radius for any power P_m is given by,

$$r = \frac{V}{2\,R_a}\sqrt{1-\frac{4\,P_mR_a}{V^2}}$$

But $\quad (P_m)_{max} = \dfrac{V^2}{4\,R_a}$, substituting above

$$r = \frac{V}{2\,R_a}\sqrt{1-\frac{P_m}{(P_m)_{max}}}$$

$$\therefore \quad r = \frac{V}{2\,R_a}\sqrt{1-m} \qquad \text{where} \quad m = \frac{P_m}{(P_m)_{max}}$$

We know, $OO' = d = \dfrac{V}{2\,R_a}$

$$\therefore \quad r = d\sqrt{1-m}$$

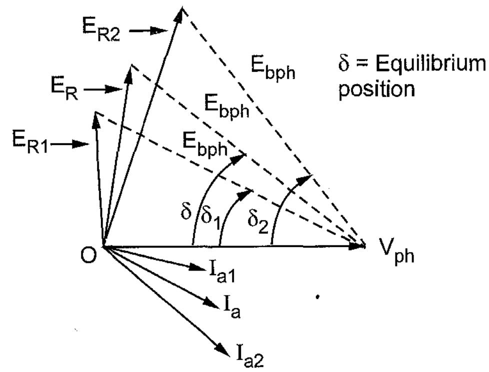
This is generalized expression for the radius for any power P_m.

HUNTING IN SYNCHRONOUS MOTOR

- When synchronous motor is on no load, the stator and rotor pole axes almost coincide with each other.
- When motor is loaded, the rotor pole axis falls back with respect to stator. The angle by which rotor retards is called load angle or angle of retardation δ.
- If the load connected to the motor is suddenly changed by a large amount, then rotor tries to retard to take its new equilibrium position.
- But due to inertia of the rotor, it cannot achieve its final position instantaneously.
- While achieving its new position due to inertia it passes beyond its final position corresponding to new load. This will produce more torque than what is demanded. This will try to reduce the load angle and rotor swings in other direction. So there is periodic swinging of the rotor on both sides of the new equilibrium position, corresponding to the load.
- Such oscillations of the rotor about its new equilibrium position, due to sudden application or removal of load is called swinging or hunting in synchronous motor.
- The main causes of hunting are,

1. Sudden change in the load.
2. Fault in the supply system.
3. Sudden change in the field current.
4. A load containing harmonic torque.

- Due to such hunting, the load angle δ changes its value about its final value.
- As δchanges, for same excitation i.e. E_{bph} the current drawn by the motor also changes.
- Hence during hunting there are changes in the current drawn by the motor which may cause problem to the other appliances connected to the same line.
- The change in armature current due to hunting is shown in the Fig.
- If such oscillations continue for longer period, there are large fluctuations in the current.
- If such variations synchronise with the natural period of oscillation of the rotor, the amplitude of the swing may become so great that motor may come out of synchronism.

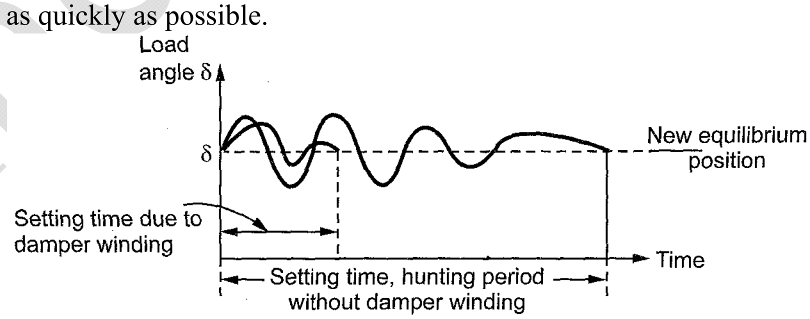


- At this instant mechanical stresses on the rotor are severe and current drawn by the motor is also very large. So motor gets subjected to large mechanical and electrical stresses.
- The various undesirable effects of hunting are,

1. It may lead to loss of synchronism.
2. It produces large mechanical stress.
3. It causes increase in losses and increases temperature rise.
4. It causes large changes in current and power flow.

Use of Damper Winding to Prevent Hunting

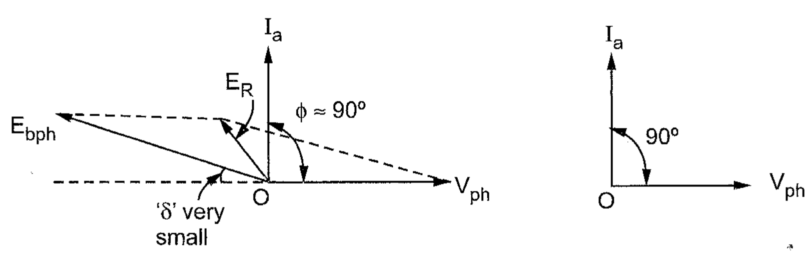
- When rotor starts oscillating i.e. when hunting starts a relative motion between damper winding and the rotating magnetic field is created.
- Due to this relative motion, e.m.f. gets induced in the damper winding.
- According to Lenz's law, the direction of induced e.m.f. is always so as to oppose the cause producing it.
- The cause is the hunting. So such induced e.m.f. oppose the hunting.
- The induced e.m.f. tries to damp the oscillations as quickly as possible.
- Thus hunting is minimised due to damper winding.
- The time required by the rotor to take its final equilibrium position after hunting is called as setting time of the rotor. If the load angle δ is plotted against time, the schematic representation of hunting can be obtained as shown in the Fig.



- It is shown in the diagram that due to damper winding the setting time of the rotor reduces considerably.

SYNCHRONOUS CONDENSERS

- When synchronous motor is over excited it takes leading p.f. current. If synchronous motor is on no load, where load angle is very small and it is over excited ($E_b > V$) then power factor angle increases almost upto 90°. And motor runs with almost zero leading power factor condition, This is shown in the phasor diagram Fig.
- This characteristics is similar to a normal capacitor which always takes leading power factor current.



- Hence over excited synchronous motor operating on no load condition is called as synchronous condenser or synchronous capacitor.
- This is the property due to which synchronous motor is used as a phase advancer or as power improvement device.

Disadvantages of Low Power Factor

The power is given by,

$$P = V I \cos\phi \quad \text{i.e.} \quad I = \frac{P}{V\cos\phi}$$

The high current due to low p.f. has following disadvantages

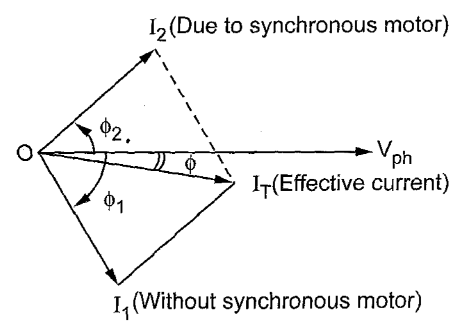
1. For higher current, conductor size required is more which increases the cost.
2. The p.f. is given by

$$\cos\phi = \frac{\text{Active power}}{\text{Apparent power}} = \frac{\text{P in kW}}{\text{S i.e. kVA rating}}$$

   Thus for fixed active power F, low p.f. demands large kVA rating alternators and transformers. This increases the cost.
3. Large current means more copper losses and poor efficiency.
4. Large current causes large voltage drops in transmission lines, alternators and other equipments. This results into poor regulation. To compensate such drop extra equipment is necessary which further increases the cost.

Use of Synchronous Condenser in Power Factor Improvement

- The low power factor increases the cost of generation, distribution and transmission of the electrical energy. Hence such low power factor needs to be corrected. Such power factor correction is possible by connecting synchronous motor across the supply and operating it on no load with over excitation.
- Now let V_{ph} is the voltage applied and I_{1ph} is the current lagging V_{ph} by angle Φ_1. This power factor Φ_1 is very low, lagging.
- The synchronous motor acting as a synchronous condenser is now connected across the same supply. This draws a leading current of $I_{2ph.}$
- The total current drawn from the supply is now phasor of I_{1ph} and I_{2ph}.
- This total current I_T now lags V_{ph} by smaller angle Φ due to which effective power factor gets improved. This is shown in the Fig.
- This is how the synchronous motor as a synchronous condenser is used to improve power factor of the combined load.



Features of Synchronous Motor

1. The synchronous motors run only at synchronous seed.
2. By varying its excitation, its power factor can be varied.
3. As it can be operated at leading power factor, it is used as a power factor correction device.
4. They are not self starting and requires an additional facility to make it self starting.
5. Under no load and over excited condition it can be used as a synchronous condenser.

Comparison Between Synchronous and Induction Motors

1. For a given frequency, the synchronous motor runs at a constant average speed whatever the load, while the speed of an induction motor falls somewhat with increase in load.
2. The synchronous motor can be operated over a wide range of power factors, both lagging and leading, but induction motor always runs with a lagging p.f. which may become very low at light loads.
3. A synchronous motor is inherently not self-starting.

4. The changes in applied voltage do not affect synchronous motor torque as much as they affect the induction motor torque. The breakdown torque of a synchronous motor varies approximately as the first power of applied voltage whereas that of an induction motor depends on the square of this voltage.
5. A d.c. excitation is required by synchronous motor but not by induction motor.
6. Synchronous motors are usually more costly and complicated than induction motors, but they are particularly attractive for low-speed drives (below 300 r.p.m.) because their power factor can always be adjusted to 1.0 and their efficiency is high. However, induction motors are excellent for speeds above 600 r.p.m.
7. Synchronous motors can be run at ultra-low speeds by using high power electronic converters which generate very low frequencies. Such motors of 10 MW range are used for driving crushers, rotary kilns and variable-speed ball mills etc.

Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service:
1. Power factor correction
2. Constant-speed, constant-load drives
3. Voltage regulation

(*a*) Power factor correction

Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors and other devices having lagging p.f. such as welders and flourescent lights etc.

(*b*) Constant-speed applications

Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc. Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

(*c*) Voltage regulation

The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor with a field regulator (for varying its excitation), this voltage rise can be controlled. When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.